A Study on Numerical Method for Analytic Continuation of Complex Function

複變函數解析延宕之數值方法研究

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Abstract

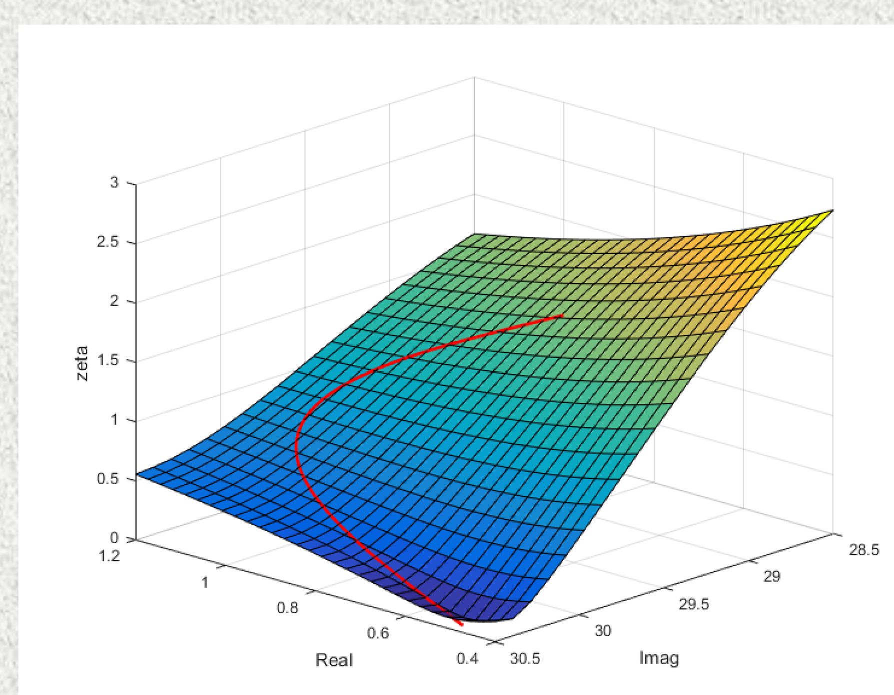
We solve the complex polynomial equations, finding the roots of Riemann zeta function $\zeta(z)$, verifying Cauchy-Riemann relations of the complex analytic functions. In this research, numerical methods are put into use.

Introduction

- The study can be divided into 3 parts.
- In the first part, the purpose is to solve the polynomial equations with complex coefficient. To do so, we take advantage of the power iteration method, a commonly used method to obtain eigenvectors or eigenvalues of a square matrix. The fact is that finding eigenvalues of a matrix is equivalent to solving a polynomial equation.
- Similar to the previous, the goal of the second part is also finding the roots with some numerical approaches; however, we focus on Riemann Zeta function $\zeta(z)$ rather than the polynomial equation. Hence the scheme applied in the former part is no longer feasible, we make use of another method called the gradient descent, an algorithm for finding the minimum(s) of the function.
- In the last part, we are going to survey the relation between the real part and the imaginary part of the complex analytic function. Given a complex analytic function, we want to investigate the values of the imaginary part when we merely have those of the real part.

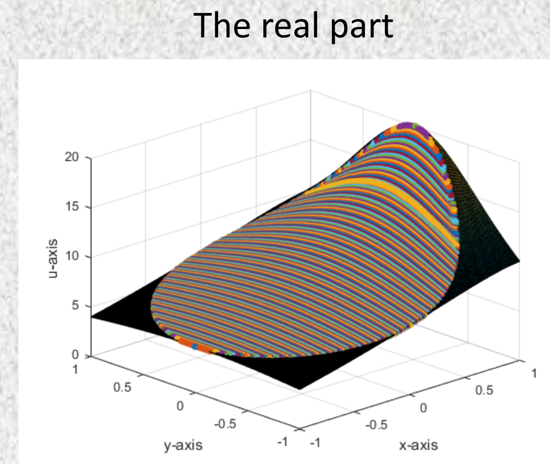
Result

- The gradient Descent

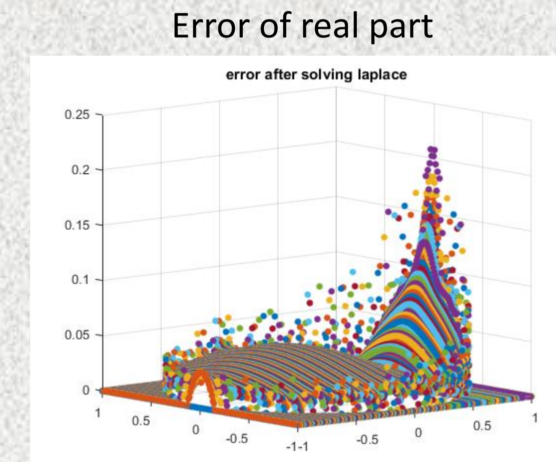


- Laplace and CR eq.

The real part



Error of real part



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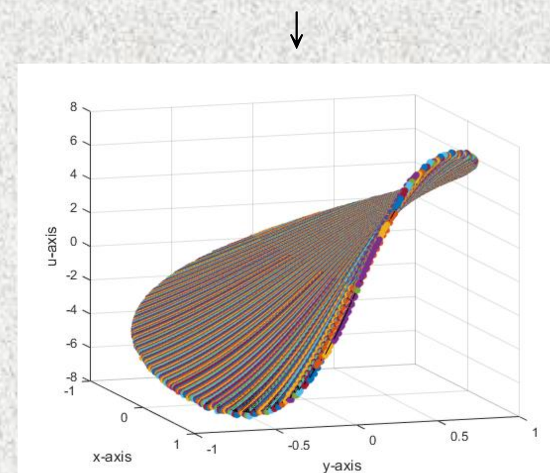
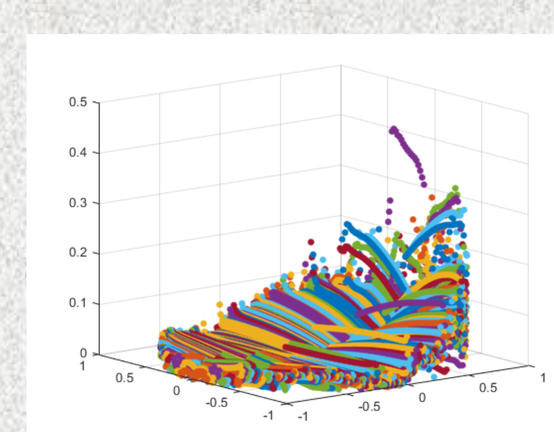


The imaginary part



Error of imaginary part

Conclusion

- In the first part we have solved the complex polynomial functions by power iteration method as well as the concept of eigenvectors and eigenvalues of matrices. However, although it seem that we can find all roots of polynomials no matter what the coefficients are, we also remain an unsolved problem.
- The second topic we have carried out an exploration of Riemann zeta function $\zeta(z)$ on the complex plane. We have successfully found several nontrivial roots of $\zeta(z)$ by making use of the gradient descent approach.
- The last part we have realized the connections between real part and imaginary part of an complex analytic function: they obey both the Cauchy-Riemann equations and the Laplace's equation. As a result, we have conducted an experiment to investigate imaginary part of the complex analytic function given its real part. Still, some numerical methods are applied.