Soliton and nonlinear Schrödinger equation

- Nonlinear Schrödinger equation:
  \[ i \frac{\partial U}{\partial z} + \frac{1}{2} \frac{\partial^2 U}{\partial t^2} + |U|^2 U = 0 \]

- Use effective approximations, such as numerical analysis, to solve nonlinear Schrödinger equation
- If the group delay dispersion effect equals to nonlinear effect, then the pulse propagates as an optical soliton
- Initial condition
  \[ U[z=0,t] = \text{sech}(t) \]
  \[ z = 0 \sim 9.4 \]
  \[ \Delta z = 0.00005 \]
  \[ t=-15 \sim 15 \]
  \[ \Delta t=0.1 \]

- Difference equation approximation:
  \[ U[z+1,t] = U[z,t] + \frac{i\Delta z}{2} \left( \frac{U[z,t+1] - 2U[z,t] + U[z,t-1]}{\Delta t^2} \right) + i\Delta z(|U[z,t]|^2 U[z,t]) \]

- Approximation with Newton-Kantorovich iteration method:
  \[ U[z+1,t] = U[z,t] + \frac{i\Delta z}{2} \left( \frac{U[z,t+1] - 2U[z,t] + U[z,t-1]}{\Delta t^2} \right) + \frac{i\Delta z}{2} \left( |U[z,t]|^2 U[z,t] + |U[z+1,t]_{\text{guess}}|^2 U[z+1,t]_{\text{guess}} \right) \]
  \[ U[z+1,t]_{\text{guess}} \leftarrow U[z+1,t] \quad \text{if} \quad \left| U[z+1,t] - U[z+1,t]_{\text{guess}} \right| > \varepsilon \]

- Approximation with discrete Fourier transform
  \[ U[z+1,t] = U[z,t] + \frac{i\Delta z}{2} (IDFT \left( -w^2 U[w] \right)) \]
  \[ + \frac{i\Delta z}{2} \left( |U[z,t]|^2 U[z,t] + |U[z+1,t]_{\text{guess}}|^2 U[z+1,t]_{\text{guess}} \right) \]
  \[ U[z+1,t]_{\text{guess}} \leftarrow U[z+1,t] \quad \text{if} \quad \left| U[z+1,t] - U[z+1,t]_{\text{guess}} \right| > \varepsilon \]

- It ought to be very careful about any pros and cons from every step of the approximation algorithms; otherwise, the simulation results turn out to be meaningless.